PHYS 101 MIDTERM October 22nd, 2007

------- WITH SOLUTIONS ------

The exam comprises two parts: 5 short-answer questions, and 4 problems. A formula sheet is attached to the back of the exam. Calculators are allowed.

Answer **all the short-answer questions** with a few words or a phrase, but be concise, please! For the problems, your grade will be calculated with the **best three problems**. Show your work.

The short answer problems are worth two points each, and the problems are worth 10 points each. Put all answers in the **red and white answer booklets** provided; you may keep this exam.

Good luck !

Short answer questions (answer all):

you should not need to do any calculations for these questions. Answer in **a few words, a short phrase, or a simple sketch.**

1) [2 pts] Can a centripetal force ever do work on an object moving in a circular path at a constant speed?

   Answer: No. The force and the displacement are always at right angles, so no work is done.

2) [2 pts] Two drivers are on a highway at the same speed, side by side. At the same instant, they each see an obstacle on the highway and start to brake (ignore reaction times). Driver 1 hits the brakes, locks the wheels, and skids to a stop. Driver 2 hits the brakes and applies the brakes to the verge of locking, but so that the wheels never lock up. Which driver stops in the shorter distance?

   Answer: Both drivers are using the force of friction to stop. The first driver is using kinetic friction (his wheels are skidding); the second is using static friction (his wheels are not skidding). $\mu_k < \mu_s$ so the second driver stops in the shorter distance.

3) [2 pts] Your professor is giving a demonstration by tying a stone to a string and whirling it in a circle at constant speed. He tries it in two different ways: in a horizontal circle, and in a vertical circle. Assuming the speed is constant and the same in both cases, in which case is the string most likely to break? Explain **briefly.**

   Answer: If the speed is constant and the same, then the radial acceleration is the same (v^2/r). In the horizontal case, the string tension is always mv^2/r. But in the case of a vertical circle, at the bottom of the circle, the string tension must balance mg and provide mv^2/r of radial force – so the tension is greater and that's where the string is most likely to break.

4) [2 pts] A sailboat is moving at constant velocity. Is work being done by a net external force acting on the boat?

   Answer: At constant velocity there is no net force, and no change in kinetic energy, so no work is being done.

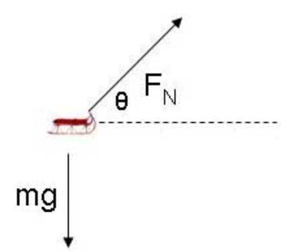
5) [2 pts] In Superman© comic books, the hero is often seen hovering in mid air and then grabbing a villain and throwing him forward. Superman remains stationary during the throw. Besides the awful clothing choices of our protagonist, what is wrong with this scenario?

   Answer: Clearly the authors have forgotten about conservation of momentum. When Superman throws the villain, that throw is internal to the system. There is no external force, so momentum of the system is conserved. If the villain is thrown to the right, Superman must recoil to the left (and vice versa).

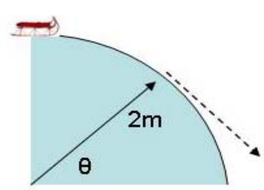
Problems (do 3 out of four):

1) [10 pts] A child is sledding on a semi-circular hill as shown, starting from rest at the top of the hill. Assume a frictionless surface. At some point down the hill the sled leaves the surface.

   a) Draw a free-body diagram for the sled at an arbitrary angle θ.



   b) When the sled leaves the surface, the normal force is zero. Calculate where this happens (ie, at what value of the angle θ). (Hint: at that instant consider the motion to be uniform circular motion).



Solution:
The net force in the radial direction must be F_N (outwards) – mg sin(θ) (inwards) which must be the radial force mv^2/r:

$$F_N - mg\sin(\theta) = -mv^2/r$$

When the sled leaves the surface, F_N becomes zero, so that moment is when

$$mg \sin(\theta) = mv^2/r$$

which gives:

$$\sin(\theta) = v^2/rg$$

Now, we can use conservation of energy to calculate v^2. At an angle θ, the sled has lost a height r-rsin(θ) from the top, so:

$$\tfrac{1}{2} mv^2 = mgr\,(1-\sin(\theta))$$

which leads to (after some algebra):

$$\sin(\theta) = 2\,(1 - \sin(\theta))$$

and thus:

$$\sin(\theta) = 2/3$$

Note that it's independent of r, m, … and everything else!

2) [10 pts] Sand is being poured into a dump truck that is sitting on a scale. The sand is being poured from a height of 2.50 m above the truck. Because of the impulse that the sand delivers to the truck, the scale shows a larger weight than the weight of the sand and truck together. If sand is being poured at a rate of 100 kg/s and we assume the sand is in freefall until it hits the truck, calculate the difference between what the scale shows and the true weight of the mass and truck.

Solution: Each particle of sand is in freefall, so it hits with a speed given by:

$$mgh = \tfrac{1}{2} mv^2$$

or:

$$v = \sqrt{(2gh)}$$

The difference between the true weight and what the scale shows will be due to the force of this sand hitting the truck and stopping:

$$F = \Delta p/\Delta t = v\,\Delta m/\Delta t$$
$$= \Delta m/\Delta t\,\sqrt{(2gh)}$$
$$= 700\text{ N}$$

3) [10 pts] A fire hose ejects a stream of water at an angle of 35^o from the horizontal and at a speed of 25.0 m/s. Assume the water behaves like a projectile (with no air resistance).

a) What is the highest fire that this hose can be used for?

Solution: This is just the maximum height reached by the water. We use:

$$v_{yf}^2 = v_{yi}^2 + 2\,a\,\Delta y$$

where $a = -g$ and $v_{yf} = 0$:

$$\Delta y = v_{yi}^2/2g = (25.0\sin(35^o))^2/2g = 10.5\text{ m}$$

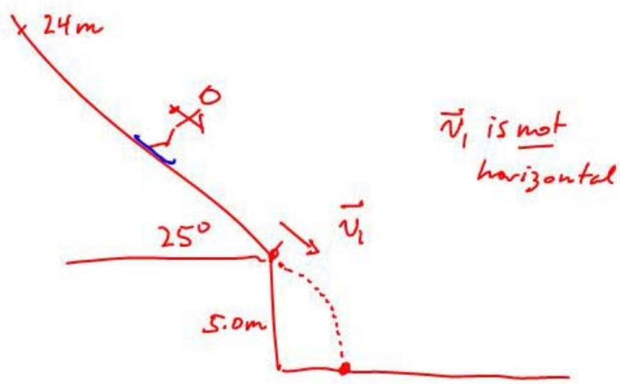
b) How far from the building should the hose be placed for that fire?

Solution: You could use the range equation and divide the range by two, or you could calculate the time for the projectile (the water) to reach the highest point of the trajectory and use that time and the horizontal speed to calculate the distance:

$$v_{yf} = 0 = v_{yi} - g\,t$$
$$t = 25.0\sin(35^o)/g = 1.46\text{ seconds}$$
$$d = 25.0\cos(35^o) \times 1.46\text{ seconds} = 30.0\text{ m}$$

4) [10 pts] An extreme skier starts from rest down a slope that has an angle of 25° from the horizontal. The coefficient of static friction between her skis and the snow is 0.22. After a distance of 24.0 m down the slope she goes over a cliff without slowing down. She lands a vertical distance 5.0 m below the cliff edge.

a) Draw a diagram. Indicate v_1 (the velocity at the cliff edge), the slope distance of 24.0 m, the vertical distance of 5.0 m, and the approximate impact point (without doing any calculations). (For extra points, colour the skis a pretty colour).



(and I really did give extra points for colour on the skis!)

b) Calculate her velocity just before she lands.

Solution: To the cliff edge, we can use conservation of energy to get her speed at that moment. Then we'll calculate the components at that instant, and do our projectile motion calculations from there.

To the cliff edge:

$$mg\,(24.0\text{ m})\sin(25^{\circ}) - (24.0\text{m})\,mg\cos(25^{\circ})\text{x}0.22 = \tfrac{1}{2}mv^2$$

(where mgcos(25) is the normal force), giving:

$$v = 10.24\text{ m/s}$$

so the x and y components at the moment the skier leaves the cliff edge are:

$$v_{xi} = 10.24\cos(25) = 9.28\text{ m/s}$$
$$v_{yi} = 10.24\sin(25) = 4.33\text{ m/s (downwards)}$$

The x component will be unchanged during the freefall, and the y component will Increase:

$$v_y^2 = v_{yi}^2 + 2g(5\text{m})$$
$$v_y = 10.8\text{ m/s}$$

So the final velocity components are v_{xi} and v_y, and the speed is:

$$v = \sqrt{(v_x^2 + v_y^2)} = 14.2\text{ m/s}$$